At this point, we have enough tools in place to do some interesting control work. We must restrict our attention to the "steady state" aspects of control situations but we can readily consider alternative steady states near the nominal operating point and use those to judge things like the need for advanced control and the potential gain available from alternate control strategies.

Our biggest limitation comes in the feedback control area. The ultimate performance of a feedback controller is only knowable through considerations of the dynamic or transient response performance. We circumvent that limitation for the moment by specifying the loop gain limit in proportional control situations. While artificial, the approach lets us get on with overall control considerations and properly focuses our attention on loop gain as an index of controllability of the process.

The key point in the sample assignment is to go through a control situation in a step by step fashion. I recommend that anyone addressing a real control situation try to formulate a "sample assignment" like this one for their situation. When you reach the point where you can do this, your understanding of the situation has reached an important milestone. Certainly, it will be necessary to augment the "problem" with transient response considerations, but by leaving them out initially, one is able to retain focus on the overall problem.

Therefore, only part of the focus in the sample assignment is on answering the questions posed. That is the minimum goal. A higher goal is to appreciate why these are the right questions to ask. In fact, since most students won't actually carry out control system designs in their careers, the most valuable knowledge will be the knowledge of what questions to ask of those specialists that actually do control system design work.

The sample assignment addresses in a cohesive way the development of a basis weight controller for a paper machine. After a review of this situation and its formal solution, students should undertake a full assignment on their own. The problem of moisture control on the papermachine serves as a convenient example. Other examples can be developed; the only requirement is that the student have sufficient understanding of the process situation to avoid having it become a "plug into formulas" exercise.
Sample Assignment on Basic Control

This is a study of basic weight control on a paper machine. Figure 1 is a P&I diagram of the situation. The following notation applies:

- BT: basis weight transmitter
- CT: consistency transmitter
- FT: flow transmitter
- FC: flow controller
- ST: speed transmitter

The relationship among paper machine variables and stock to the machine can be represented as:

\[ BW \times SM \times T = 600FS \times CS \]

where \( BW \) is basis weight in g/m²; \( SM \) is speed in m/min; \( T \) is trim in meters; \( FS \) is stock flow in 1/s; \( CS \) is consistency in %. Suppose the nominal situation is: \( BW=50; \ SM=500; \ T=5; \ FS=69.4; \ CS=3 \).

1. Make a functional representation of the process suitable for investigations of basis weight control. Treat stock flow as the manipulated variable, basis weight as the response, and consistency and machine speed as disturbances.

2. Supposing stock flow to the machine is kept constant, but consistency and machine speed are subject to variations as shown below. Estimate the range of basis weights that will occur.

<table>
<thead>
<tr>
<th></th>
<th>average</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS</td>
<td>3</td>
<td>2.7</td>
<td>3.3</td>
</tr>
<tr>
<td>SM</td>
<td>500</td>
<td>450</td>
<td>550</td>
</tr>
</tbody>
</table>

If the basis weight specification on the paper is 45<BW<55, is there a need for a better control strategy?

3. Develop a functional representation of the process with a feed forward control system in place. Assume that the stock flow rate can be manipulated by the control system, and that both consistency and machine speed can be measured. Be sure that your controller has an input for desired basis weight.

4. Develop an exact algorithm for the feedforward controller shown in part 3.

5. Develop a linear incremental model of the process in the form:

\[ BW = K1 \times FS + K2 \times SM + K3 \times CS \]

where \( BW, FS, SM, \) and \( CS \) are variations about the nominal conditions. The simplest approach is to conduct "thought experiments" on the process using the fundamental fiber balance shown at the beginning of this sheet, processing the "data" to obtain the coefficients \( K1, K2, K3 \).

6. Show a block diagram representation of the process based on the linear incremental model, again treating \( FS \) as the manipulated variable, \( BW \) as the response, and \( CS \) and \( SM \) as disturbances.

7. Repeat the analysis of disturbances carried out in part 2 above, using the linear incremental model to obtain the expected range of variation of basis weight. Compare your results with the "exact" method used before. Comment on the suitability of using the linear incremental model.

8. Using the linear incremental process model, develop a block diagram representation of the process with a feedforward control system as previously represented by the functional diagram in part 3. Designate the tunable parameters of the feedforward control system with the notation \( KF1, KF2, KF3 \).

9. Make a functional representation as a single block of the process with the control system in place.
10. By "reading" the block diagram of part 8, and with reference to the functional diagram of part 9, make an algebraic representation of the controlled process in terms of the incremental variables and the undetermined controller constants KFBW, KFCS, KFSM.

11. Use the results of part 10 to determine the tuning of the controller parameters KFBW, KFCS, KFSM, which will produce "perfect control".

12. Using the controller portion of the block diagram of part 8, determine what stock flow will be going to the machine if the desired basis weight is 50, the machine speed is 460, and the consistency is 3.2. Take that stock flow and the other conditions and apply the fiber balance formula presented at the beginning of this assignment to determine how close the control would be to perfect.

13. omitted

14. An alternate approach to basis weight control would be to rely on feedback control using the measured basis weight. Make a functional representation of the process with such a scheme in place. Consistency and machine speed will remain as disturbances in the process, but will not be considered in the control system. Of course, there will still need to be a desired basis weight input.

15. Make a block diagram corresponding to the functional diagram of part 14. Use the process block diagram used in part 6 and assume the controller is a proportional controller with gain Kc.

16. Determine the controller gain required to obtain a loop gain having a magnitude of 4. Pay careful attention to obtaining the correct sign for the controller gain (guarantee an odd number of minus signs around the loop, or guarantee that the control action tends to reduce the error).

17. Using the controller gain found in part 16 and the disturbances in part 2 estimate the range of variation in basis weight with this control system. With this gain, what target basis weight could be used; what annual savings would be achieved compared to running at the nominal weight. (Assume that paper machine furnish costs $500 per metric ton and that all quality specifications other than basis weight are being easily met.)

18. Compare the controller found in part 16 with our ideal of "perfect control". Can a proportional controller achieve that ideal?

19. What would you recommend as the correct structure for basis weight control? (a) feedforward; (b) feedback; (c) other (specify). Discuss your recommendation.
Figure 1. P & I diagram of paper machine for sample assignment.

BT ≡ basis weight (g/m²) transmitter
CT ≡ consistency transmitter (g fiber/g stock)
SOLUTION TO THE SAMPLE ASSIGNMENT

1. Make a functional representation of the process. The functional representation corresponds to the function we obtain by solving the fiber balance formula for basis weight in terms of stock flow, consistency, and machine speed.

\[ BW' = \frac{600FS' CS'}{SM'T'} \]

Trim should be substituted in to obtain the revised formula

\[ BW' = \frac{120FS' CS'}{SM'} \]

And the generic form can be shown as

\[ BW' = f( FS', CS', SM') \]

from which the functional diagram shown in Figure 1 follows.

![Functional Diagram](image)

Figure 1. Functional Diagram of a Paper Machine

Note that these are actual values in Figure 1 and should therefore carry a prime to be absolutely correct. I think, however the meaning is clear enough in the diagram that we need not carry this extra notation.

2. Estimate the range of basis weight variations. In this part, we are studying variability in the process with "no control". Since operators invariably do exercise control, this is just a coarse cut at understanding the disturbances and possible consequences.

We are given average values for consistency and speed. We assume that stock flow stays at its nominal value of 69.4 liters per second. Since the average consistency and speed are the same as the nominal values, we know that the average basis weight is 50 g/m².

Now, both speed and consistency vary from specified minimum to maximum values. However, we should understand that the actual values at any point in time are random variables, i.e. low speed could
go with high consistency or visa versa. Also, we assume that any pair of values may arise and retain their values long enough for the process to come to steady state. Since we are interested in finding the minimum and maximum basis weights that can arise, we need to think about worst case situations.

Think first about a worst case light-weight-situation. At constant flow, that corresponds to a low consistency: 2.7%. Now, staying with worst case light paper, what about speed: at constant flow, the paper is lightest when the speed is highest. So, the minimum basis weight will occur when the consistency is 2.7% and the speed is 550 meters per second. The resulting weight is found from the formula as:

\[ BW' = 120*69.4*2.7/550 = 40.9 \text{ g/m}^2 \]

Without agonizing, we should be able to conclude that the maximum weight will occur when consistency is 3.3% and speed is 450, when the formula gives \( BW' = 61.1 \). So we conclude that the "no control" weight range on the machine is:

\[ 40.9 < BW' < 61.1 \]

By this analysis, the product specifications cannot be met without a control strategy better than "no control".

Some further discussion is justified. First, one can find situations where there is some interest in improving control but where an analysis like this will suggest that the disturbances don't seem severe enough to justify control. In at least some such cases, that is the correct answer, and one may find later that the impetus for the idea of control came from outside the plant, either from a vendor or from casual conversations at a conference where another plant with different conditions was discussed. However, the failure to show variability in this manner may also mean that not enough is known about the disturbances in the situation. One usually does a direct statistical analysis on the variable to be controlled: basis weight in this case.

Second, we considered a situation where the disturbances had equal variation above and below the mean. That is not invariably the case, so both extremes should be independently worked out as was done here. In this case, even though the disturbances were subject to equal high and low excursions, the basis weight variations are not uniformly spaced about the mean of 50. That is due to the nonlinearity in the problem, specifically the fact that weight is inversely related to speed.

Third, there may be some concern about the quality of our statistics. People who are statistics buffs would normally want to assume a normal distribution, give the standard deviations of consistency and speed and ask about the standard deviation of basis weight. That problem can be solved readily for linear incremental models but I consider it too technical for our present purposes. That problem can only be solved approximately for nonlinear models such as we are presently using.
The advantage of using the normal distribution is that along with the size of the extreme variations, one gets some information about the frequency of occurrence of extreme situations. We have not specified anything about the distribution of the consistency and speed variations except the extreme points. Therefore, it is not possible to say how often one might see a low consistency and high speed situation simultaneously. Certainly it would be surprising if it were an every day thing.

The range we computed here is just a guideline; the computations are easy and force you to think clearly about the problem. My overall concept is that if you need really fancy statistics to decide on the worth of a control system, it might be better to work on a different control problem where the gains are easier to prove. Our rough approach will be quite good enough for starters most situations of practical importance.

3. Functional representation of process and feedforward control system. From the description, we know that our controller will be describable by the following functional equation

\[ FS' = g( BWSP', CS', SM') \]

that is, stock flow rate will be determined from the setpoint on basis weight, the actual consistency, and the machine speed. The controller can be represented by the functional diagram shown in Figure 2 and we can show the whole situation by linking the controller with the process as shown in Figure 1 giving the answer to this part as in Figure 3.
4. Exact algorithm. An algorithm is a formula or rule for taking the three inputs, BWSP’, CS’, and SM’ and generating the output, FS’. This is one of the rare cases where we can easily generate an exact algorithm; we solve the fiber balance given in the problem for stock flow in terms of basis weight, consistency, and speed, and substitute the basis weight setpoint for the basis weight obtaining:

\[ FS' = \frac{BWSP' \times SM'}{120 \times CS'} \]

The controller block in the diagram of Figure 3 must be programmed to implement this formula. With modern DCS equipment, that can be easily done.

5. Develop a linear incremental model using "thought experiments". We have one "data point" to start with. We need a minimum of three additional data points to enable us to develop the three coefficients in the linear incremental model. The simplest set of experiments involve holding two things constant and varying the third. The data reduction effort is minimized with this approach. Consider the following table as the definition of the experiment.

<table>
<thead>
<tr>
<th>FS’</th>
<th>SM’</th>
<th>CS’</th>
<th>BW’</th>
</tr>
</thead>
<tbody>
<tr>
<td>69.4</td>
<td>500</td>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>75</td>
<td>500</td>
<td>3</td>
<td>54</td>
</tr>
<tr>
<td>69.4</td>
<td>550</td>
<td>3</td>
<td>45.5</td>
</tr>
<tr>
<td>69.4</td>
<td>500</td>
<td>3.1</td>
<td>53.3</td>
</tr>
</tbody>
</table>

As shown, the experiments could be conducted on the machine as well as in "thought" form. The table below uses the fiber balance to present the results of the experiment.

<table>
<thead>
<tr>
<th>FS’</th>
<th>SM’</th>
<th>CS’</th>
<th>BW’</th>
</tr>
</thead>
<tbody>
<tr>
<td>69.4</td>
<td>500</td>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>75</td>
<td>500</td>
<td>3</td>
<td>54</td>
</tr>
<tr>
<td>69.4</td>
<td>550</td>
<td>3</td>
<td>45.5</td>
</tr>
<tr>
<td>69.4</td>
<td>500</td>
<td>3.1</td>
<td>53.3</td>
</tr>
</tbody>
</table>

From the results, we can obtain the coefficients. For the K₁, the stock flow coefficient, we find

\[ K_1 = \frac{(54 - 50)/(75 - 69.4) = .71} \]

\[ K_2 = \frac{(45.5 - 50)/(550 - 500) = -.09} \]
\[ K_3 = \frac{(53.3 - 50)}{(3.2 - 3)} = 16.5 \]

So, our linear incremental model of the process is:

\[ BW = 0.71 \text{FS} - 0.09 \text{SM} + 16.5 \text{CS} \]

We have now shown the formula with the notation we will use for incremental variables. The formula gives the change in basis weight arising when there is a change in flow, speed, and/or consistency. Some students take a more complicated view of the problem of determining the coefficients, seeing this as the problem of solving three equations for the three unknown coefficients, K1, K2, and K3. Here, the experiments were designed so that the nominal conditions and one experimental condition uniquely defined one coefficient. Also, carefully examine the way the influence coefficients are computed:

\[ K = \frac{\text{test weight} - \text{nominal weight}}{\text{test value} - \text{nominal value}} \]

6. Block diagram of the process. Key off the functional diagram in Figure 1, showing weight on the right and stock flow on the left with the disturbances coming in from the top. Since there are three terms in the linear incremental model, we need three blocks and two summers, as shown in Figure 4.

![Figure 4. Block Diagram of a Papermachine](image)

7. Redo the variations calculations using the linear incremental model. We still know the worst case situations: \( C' = 2.7, S' = 550 \) and \( C' = 3.3, S' = 450 \). In terms of variations, these are \( C = -0.3, S = +50 \), and \( C = +0.3, S = -50 \). So, we find the low weight variation to be:

\[ BW = 16.5*(-0.3) - 0.09*(50) = -9.5 \]
and for the high weight variation,

\[ BW = 16.5 \times 0.3 - 0.09 \times (-50) = +9.5 \]

Since \( W' = W + W_n \), the range of weight variation is:

\[ 40.5 < BW' < 59.5 \]

according to the linear incremental model. The model gives a bit smaller range of variation than we found previously (40.5 to 61.1). Also, since our disturbance variations were symmetric about their mean values, the linear model gives a weight variation range which is symmetric about 50 g/m². We still find that the process will go way outside the product specification range unless control is exercised.

8. Develop the block diagram of the process with feed forward control in place. We can produce this from the functional diagram in Figure 3. Since the controller generates FS given BWSP, CS, and SM, the linear incremental model of the controller must be:

\[ FS = KF_1 \times BWSP + KF_2 \times SM + KF_3 \times CS \]

and we will need three blocks and two summers. The total block diagram is given in Figure 5.

![Block diagram for feedforward controller and papermachine](image)

9. Overview functional diagram. Here we are wrapping a big block around the diagram in Figure 2. We will still have BW’ going out on the right, and CS’ and SM’ coming in the top. FS’ will be "buried"
somewhere inside the block. BWSP′ will be coming in from the left as the process operator's adjustment once the control system becomes part of the paper machine. The overview functional diagram is given in Figure 6.

Figure 6. Overview of functional diagram of papermachine with an imbedded basis weight control system.

10. Read the functional diagram in Figure 5 and develop the linear incremental model of the process with the controller in place. The best way to approach this is to make use of the key property of linear models: linear superposition. This concept tells us that we can work with the linear model one variable at a time, to obtain a series of one variable models of the process. We can then find the multiple variable model of the process as the sum of the one variable models. We begin with the consistency variation input. There are two paths from consistency to basis weight; we find the combined influence coefficient to be:

\[ BW = (16.5 + KF_3 \times 0.71) \times CS \]

This is the first of three such "one variable" models. This model describes the variation in BW if SM and BWSP are zero, equivalent to holding SM′ and BWSP′ at their nominal values of 500 and 50. Similarly, we find for speed,

\[ BW = (-0.09 + KF_2 \times 0.71) \times SM \]

which is true provided CS and BWSP are zero. Finally, there being only one path from BWSP to BW, we find,

\[ BW = KF_1 \times BWSP \]

which is exact provided CS and SM are zero (CS′=3; SM′=500).
Now, to get a single model encompassing simultaneous variations in all three inputs, we add the individual contributions to get:

\[
BW = KF_1*0.71*BWSP + (-0.09 + KF_2*0.71)*SM + (16.5 + KF_3*0.71)*CS
\]

This is the formula we need. It works for any values of BWSP, SM, and CS but is subject to errors if the implied deviations from BWSP_n, SM_n, and CS_n are large.

11. Tune the controller for "perfect control". If the control were perfect, when the operator asked for a change in basis weight of B pounds, the control system would change flow rate by the appropriate amount. If we assume that SM and CS are zero (no change in speed or consistency),

\[
BW = KF_1*0.71*B
\]

and if we further express our "expectation" that the weight will actually change by B pounds we have

\[
B = KF_1*0.71*B
\]

which can be solved for KF1 as:

\[
KF_1 = 1/0.71 = 1.41
\]

This makes the controller "perfect" with respect to setpoint changes. It does nothing for consistency variations or speed variations, but neither does it inhibit in any way our opportunity to achieve perfection with respect to those variations since we still have free choice of KF2 and KF3.

For speed variations, "perfect" means that when speed changes, there is no change in basis weight. We can see if a tuning will achieve this by assuming that CS and BWSP are zero and investigating the case where only speed changes. Thus,

\[
BW = (-0.09 + KF_2*0.71)*SM
\]

We must arrange for BW to be zero no matter what the value of S is. This can be done provided we can cause the influence coefficient, represented in the parentheses to be zero. We need to find a value for KF2 such that

\[
0 = -0.09 + KF_2*0.71
\]

This can be solved for KF2 as

\[
KF_2 = 0.09/0.71 = .127
\]
At this point the controller will work "perfectly" for speed and setpoint variations provided $KF_1 = 1.41$ and $KF_2 = 0.127$. We still have the opportunity to set $KF_3$ to make it perfect with respect to consistency variations. This will be so if

$$BW = (16.5 + KF_3 \times 0.71) \times CS = 0$$

for any value of CS. Again, the thing in parentheses must be zero for this to happen.

$$0 = 16.5 + KF_3 \times 0.71$$

which can be solved for $KF_3$

$$KF_3 = -16.5 / 0.71 = -23.2$$

We can now combine our separately determined results (again using "linear superposition") to find the controller equation

$$FS = 1.41 \times BW_{SP} + 0.127 \times SM - 23.2 \times CS$$

which quantitatively gives the flow value the controller will use for any combination of operator command, speed change, and consistency change. We can also see by inspecting the sign of the coefficients that it will add stock if the operator wants more weight or if the speed goes up. It will take stock off if the consistency goes up.

12. Check the controller for the following conditions: $BW_{SP} = 50$; $SM = 460$; $CS = 3.2$. To do this we first determine the variations in the three quantities:

$$BW_{SP} = BW_{SP}' - BW_{SP} = 50 - 50 = 0$$

$$SM = S' - S = 460 - 500 = -40$$

$$CS = CS' - CS = 3.2 - 3 = 0.2$$

Then we find the $FS$ the controller will use as

$$FS = 1.41 \times 0 + 0.127 \times (-40) - 23.2 \times 0.2 = -9.72$$

We can check our work so far by plugging $FS$, $SM$, and $CS$ into the process linear incremental model to determine the change in $BW$ as

$$BW = 0.71 \times (-9.72) - 0.09 \times (-40) + 16.5 \times 0.2 = -0.0012$$
This is such a small change that we can safely ascribe it to round off error in our arithmetic work, and conclude that the controller is incrementally perfect.

The problem is more concerned with a verification more like we would get if we were trying it on the machine. To do that, we need to compute \( FS' \) and substitute it, along with \( SM' \) and \( CS' \) into the fiber balance to estimate actual machine conditions. Thus,

\[
FS' = FS + FS_n = -9.72 + 69.4 = 59.7
\]

and the fiber balance dictates that

\[
BW' = 120*59.7*3.2/460 = 49.8
\]

Since we are trying for 50, there is an error of 0.4%; good enough for government work.

14. Feedback control: functional diagram. The feedback controller takes an operator setpoint and the measured weight in and produces a stock flow value out. The controller alone can be as shown in Figure 7. To this we must add the functional diagram of the process given in Figure 1 to get the situation on the process when we use a feedback controller for basis weight control as in Figure 8.

![Figure 7. Functional diagram of feedback controller](image1)

![Figure 8. Functional diagram of papermachine with feedback control.](image2)
Notice that although the consistency and speed disturbances are entering the functional diagram, the controller has no direct knowledge of them.

15. Block diagram of the feedback control situation with a proportional feedback controller. Application of the general form of a proportional controller to this situation gives:

\[ FS = K_c \times (BWSP - BW) \]

where the basic idea is to form the difference between the setpoint and the measured value and multiply the result by the controller gain. This can be represented on a block diagram as in Figure 9.

Figure 9. Block diagram of a feedback controller.

We have used the special form of the summer, with the minus sign located at the point where the measured value enters to represent the computation of the difference between setpoint and measurement that is characteristic of feedback controllers. This diagram can be combined with the block diagram of the process given in Figure 4 to give a complete picture of the situation as in Figure 10.
16. Tune the controller for a loop gain of 4. By inspecting the block diagram of Figure 10, we see that the loop gain in question is:

$$\text{loop gain} = K_c \times 0.71$$

We can get the sign of $K_c$ right by using the rule of thumb that there must be an odd number of minus signs as one traverses the loop. The only negative sign here arises as the loop passes through the summer; one negative sign is an odd number of negative signs so we can assume that $K_c$ should take on a positive value. Now, to obtain a loop gain of 4, we must have

$$4 = K_c \times 0.71$$

which can be solved to yield

$$K_c = 4 / 0.71 = 5.6$$

We have tuned for a specified loop gain. Since loop gain is a dimensionless quantity, we can use a rule like this and it makes good sense. As it will turn out, 4 is a large loop gain; we will often have to settle for less. Fortunately, we will be able to use more elaborate controllers than proportional controllers to
offset the deficiency introduced by a small loop gain. Here, we should do pretty well with proportional control.

The question of setting the sign of the controller gain is a very important one. The "odd number of minus signs" rule is not a bad idea. It is also possible to get it right by thinking clearly. What you must decide in each situation is what direction you want the manipulated variable to move if the error is positive. In our case, a positive error implies that the setpoint is higher than the actual weight, and that means we want to add stock to bring the weight up. So a positive error implies a positive output of the controller: Kc must be positive. We will find as many situations in the world where Kc needs to be negative as positive.

The consequences of getting the sign of Kc wrong are terrible. If we were to use a negative Kc here, if the operator tries to increase the weight, the controller will lower the stock flow. If you allow yourself to consider subsequent actions, you will conclude that the controller will eventually shut off stock altogether. Proper analysis of such a situation requires the use of transient analysis techniques, but the qualitative conclusion here is correct.

17. Estimate performance of the feedback controller with tuning above. The block diagram of Figure 10 is our entry to this problem. For each disturbance, we can determine the influence coefficient with the control system in place using the basic rule for feedback situations:

\[
\text{closed loop gain} = \frac{\text{forward path gain}}{1 + \text{loop gain}}
\]

We have tuned our controller so the loop gain is 4, so the denominator in this formula is 5. By inspection of Figure 10, we can see that there is only one path from the disturbances to the basis weight and that the forward path gain is the same as the original process influence coefficient. Therefore, with the control system in place and tuned to the specification here, the influence coefficient relating each disturbance to the weight will be one fifth the value it had without control. Therefore, our worst case variations will be reduced by a factor of 5. We previously decided that the variation in basis weight would be plus or minus 9.5 pounds. Thus with the controller in place we expect the variations to be 9.5/5= plus or minus 1.9 pounds, or that basis weight will be

\[
48.1 < BW < 51.9
\]

if the setpoint is 50.

Since our specification for weight accepts weights as low as 45, and out light end variation is -1.9, we could set the setpoint to as low as 45+1.9=46.9 and still meet the light end specification. A metric ton of paper corresponds to an area of

\[
\text{area per nominal ton} = 1000*1000/50 = 20,000 \text{ square meters}
\]
We will be using

\[ \text{new fiber usage} = 46.9 \times \frac{20000}{1000/1000}, \]

so the fiber savings are

\[ \text{savings} = (50 - 46.9) \times \frac{20000}{1000/1000} = 0.062 \text{ tons of fiber per nominal ton of paper}. \]

Our dollar savings are $500 \times 0.062 = $31 per nominal ton of production. The machine produces paper at the rate of 500 meters per minute with a trim of 5 meters; 2500 square meters per minute; about 64,800 nominal tons per year. We could project annual savings of $31 per ton for 64,800 tons or about two million dollars of annual savings.

We have already discussed how one handles justification. When you can reasonably project savings as large as this, you know your project will be approved. The thing to do is to put a safety factor into your calculation. Instead of projecting a target weight of 46.9 on the 50 g/m² grade, project 48.5, cutting your savings in half but still leaving lots of justification. If the loop gain of 4 does not materialize, there is still room to meet the proposed spec; if other factors prevent one from lowering the setpoint as far as the original spec, you still will have a successful control project.

18. Is the feedback controller a "perfect" controller. We already know that the basis weight will vary with this controller if the disturbances vary, so it is not perfect. The variations are only one fifth those with "no control" so it is good, not perfect. We should also look at the setpoint performance. Again using our rule for reading feedback diagrams, we decide that

\[ \text{BW} = \frac{0.71 \times Kc}{1 + 0.71 \times Kc} = \frac{4}{5} = 0.8 \]

When the operator asks for a one pound change, she will get only 0.8 pounds. This may be noticeable; we should suggest that the operators multiply their changes by 1.2 before entering them if they want precise control. This controller is good but not perfect. The loop gain limitation, about which we have only heard so far, prevents a proportional controller from being "perfect".

19. Recommendation. It appears that the feedforward controller outperforms the feedback controller, since it actually seems to achieve perfection. The weak link with the feedforward controller will be the measurement of consistency. The readings we get could be rather different from the actual consistency; if that happens, the basis weight will also deviate and the operators will have to correct the setpoint frequently. On the other hand, the adjustments for setpoint changes and speed changes are likely to be quite good.
The feedback controller is not dependent on information from the consistency controller; its only limitation is in the loop gain limit, which results in 20% of the disturbance getting through in this case. One could wish for an ideal world in which we could combine the feedforward and feedback controllers. We would only implement the speed and setpoint inputs of the feedforward controller, leaving the remaining variations due to consistency to be taken on by the feedback controller.

**EXERCISE** Design a feedforward/feedback controller for basis weight where speed changes and setpoint changes are handled with feedforward control and consistency changes are handled with feedback control.

**ASSIGNMENT** Show that if the feedforward controller succeeds perfectly with respect to speed but does not compensate for consistency variations at all (as if the sensor fails to register the real changes) then the performance of the feedforward controller for the variations given in part 2 will be larger than for the feedback controller with a loop gain of 4.