

## Evaluating the Growth Response of a Forest Stand to Fertilization in the Absence of Replication

Investigations in many scientific areas make use of non-replicated observations or trials. This is particularly true when the scope of the experiment is large or the expense of achieving replication becomes prohibitive. The classic forestry application is where experiments are performed on a large scale such as where one watershed receives a treatment and the outcome is compared with that of a nearby untreated one. (e.g. Bormann and Likens 1979). It can be argued very effectively that a single observation does not provide an adequate basis for inference. However Abbott and Rosenblatt (1963) have demonstrated conclusively that it is possible to construct confidence intervals for the midpoint of a known distribution from a single observation. Furnival et al. (1989) reported an improved procedure that yields shorter confidence intervals and Valentine et al (1991) presented a convincing demonstration of the improved procedure. The Stand Management Cooperative (SMC) installed a series of 13 operational thinning and fertilization trials in the Gifford Pinchot National Forest in 1976 in order to evaluate stand responses to various silvicultural regimes. The installations covered a large range of site types and stand composition. However, due to operational constraints, very limited on-site replication of treatments was permitted. In order to determine if there was in fact a response to the fertilization treatment the method presented by Valentine et al. (1991) was utilized. Analysis of the data from one of the installations is presented.

### Field Procedure

The installation was established in a stand that was 18 years old from seed. The stand was thinned to 400 trees per acre and two 0.2 acre plots were established. One plot was fertilized with 200 lbs N per acre applied as urea and refertilized at ten and at 16 years after establishment. The other plot was untreated except for the initial thinning. The trees on the plots were measured for height and diameter every two years for 20 years. The stand has an average elevation of 2400 feet with a 35 percent slope and a north eastern aspect (45°). The stand also has a site index of 95 feet at breast height age 50.

The collected data was checked and entered into the SMC database system. A plot level summarization program was written and utilized to generate relative density, stems per acre, quadratic mean diameter, basal area, average height of the 40 largest trees per acre, and plot volumes.

### Analysis

To determine if there was in fact a significant response to the fertilization treatment it is necessary to generate an a priori estimate of what was expected to happen in the absence of a fertilization treatment. A method employed by McWilliams and Burke (1994) in a different but related context is to make use of a growth model to simulate growth of a plot with identical initial conditions as the

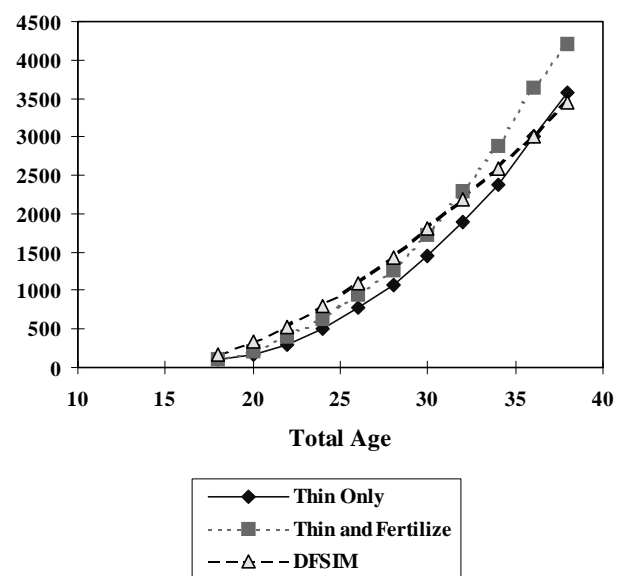


Figure 1: Volume estimates over time for installation 307, ages 18-38 with unfertilized growth of fertilized plot simulated with DFSIM

fertilized plot, but simply omitting the fertilization treatment. In the present case, the simulator DFSIM was used to generate stand level estimates for the various ages at which the stand was in fact measured. To do this DFSIM was run making use of the site index at 95 and assuming that the stand was planted with 800 stems per acre and thinned at age 18 to 400 stems per acre. An assumption was made

Total Age (yrs)	Thin Only (ft <sup>3</sup> .ac <sup>-1</sup> )	Thin and Fertilize (ft <sup>3</sup> .ac <sup>-1</sup> )	DFSIM (ft <sup>3</sup> .ac <sup>-1</sup> )
18	95	104	178
20	174	217	326
22	291	392	534
24	498	634	791
26	777	950	1089
28	1065	1257	1427
30	1442	1734	1799
32	1888	2284	2190
34	2371	2875	2596
36	3011	3648	3015
38	3578	4200	3442

$\mu$                        $\mu_0$                        $y$

Table 1: Age - Volume data observed on two plots and simulated by DFSIM

that the deviations between the data collected from the unfertilized plot and a DFSIM estimate would be normal.

Figure 1 contains the volume estimates from the DFSIM run and the plot level measurements from the plot summaries. As can be seen, there is a reasonable level of agreement between the DFSIM volume estimate and the thinned plot volume estimate, while there is a difference between the fertilized plot and the unfertilized plot as well as the DFSIM estimate of volume. Table 1 contains the data used to create Figure 1.

To determine if there is a significant difference in the volumes between the fertilized and the unfertilized plots the test is:

$$\text{Equation 1: } \mu_0 > (y + \mu_1) / 2 + k | y - \mu_1 |$$

where :  $\mu_0$  is the observed result  
 $y$  is the a priori estimate  
 $k$  is the expansion factor for selected a and distribution of error  
and  $\mu_1$  the observed untreated result.

Assuming a normal distribution and selecting an a level of 0.20 gives a  $k$  value of 2.31 (Valentine et al. 1991).

Substituting values from Table 1 at age 38 into equation 1 gives the relationship:

$$4200 > 3510 + 314.16$$

which being true indicates that the fertilization resulted in a significant increase in the volume production of this stand by total age 38, unless a one in five chance has occurred.

**Discussion**

The method of analysis presented illustrates how some inferences can be made from unreplicated trials. Replication in trials is preferable in terms of statistical confidence in the results. However replication can be lost as when some plots are destroyed by fire or storm damage. In such cases, this method allows some inferences to be salvaged from a study that would otherwise have to be completely written off. It also allows some inferences to be made when replication is impossible to achieve because of cost or available area. However, it must be emphasized that using this method forces upon the investigator a higher Type I error rate than is customary.

**Literature Cited**

Abbott, J.H. and J.I. Rosenblatt. 1963. Two stage estimation with one observation of the first stage. *Ann. Inst. Stat. Math.* (Tokyo). 4:229-235.

Bormann, F.H. and G.E. Likens. 1979. *Pattern and process in a forested ecosystem.* Springer-Verlag, New York. 253p.

Furnival, G.M., T.G. Gregoire, H.T. Valentine. 1989. Confidence intervals and significance tests for a single trial. *Comm. Stat. Theory Meth.* 18(10): 3749-3761.

McWilliams, E.R.G. and T.E. Burk. 1994. Evaluation of eight forest fertilizer response estimators by means of a simulation study. *Can. J. For. Res.* 24:107-119.

Valentine, H.T., G.M. Furnival, and T.G. Gregoire. 1991. Confidence intervals from single observations in forest research. *For. Sci.* 37 (1): 370-373.

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